Journal of Applied Mechanics and Technical Physics, Vol. 38, No. 2, 1997

ELECTROMAGNETIC FIELD AND CURRENT WAVES GENERATED BY A SHOCK WAVE ENTERING A CONDUCTOR WITH A TRANSVERSE MAGNETIC FIELD

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UDC 539.63

Shock Wave in a Conducting Material as a Current Generator. Bichenkov [1] showed that a current with linear density

$$i_s = \frac{c}{4\pi}B_0(n-1) \tag{1}$$

is generated when a steady shock wave propagates in a conductor with a transverse magnetic field. Here B_0 is the field ahead of the wave front and $n = \rho_f/\rho_0$ is the compression of the material. The distribution of electric and magnetic fields arising in the material leads to the propagation of the current wave together with the shock-wave front. In this case, for the wave pattern to be stationary, it is necessary that the magnetic field behind the wave front be uniform and equal to its "frozen" value $B_f = nB_0$, i.e., there are no currents behind the shock under stationary shock propagation. Ahead of the shock, a stationary transient layer into which the magnetic field diffuses forms, and a corresponding current distribution occurs. It was unexpected that the thickness of this layer is completely independent of the state of the material behind the shock and is determined only by the conductivity of the starting material. The thickness is defined as

$$l_0 = \frac{c}{4\pi\sigma_0}\frac{c}{D}.$$

We call this layer a diffusion layer, and l_0 is called the diffusion thickness of the current layer. The expression for the thickness includes the electric-charge relaxation time in a material with conductivity σ:

$$\tau_r = c/(4\pi\sigma). \tag{2}$$

In addition to the diffusion region, a portion of the current can be localized inside the shock-wave front. This portion of current depends mainly on the ratio of the shock-front thickness l_w to the diffusion thickness l_0 and increases with an increase in l_w.

In hydrodynamics, the first fruitful approximation is the idealization of a shock wave by a structureless discontinuity with zeroth shock-front thickness. Assuming that the forces producing an ideal hydrodynamic discontinuity act only on the ionic base of the conductor and considering the interaction between the electron and the ionic components within the two-liquid MHD-model, in which the friction between the components is determined only by the conductivity of the material, we established that the density of the electron component relaxes to the ionic density as a result of multiple oscillations with the so-called plasma frequency $\omega_p = \sqrt{4\pi n_e e^2/m}$. Here n_e is the electron density per unit volume and e and m are the charge and mass of an electron. In this case, $Q = \omega_p\tau_0$ oscillations occur in the time of electron-momentum diffusion by ions τ_0, i.e., one can consider Q as a peculiar measure of the "quality factor" of the material. Since the time τ_0 determines conductivity, one can estimate Q: $Q = 14$ for Ti and 500 for Al.

The time during which the equilibrium density of the electron liquid is attained can be estimated as $\tau_{ei} \simeq 2Q^2(1+s/n^2)\tau_r$, where τ_r is the relaxation time of the electric charge in the conducting material (2) and $s = \sigma_f/\sigma_0$ is the conductivity jump of the material in the shock wave. This implies that for conductors such

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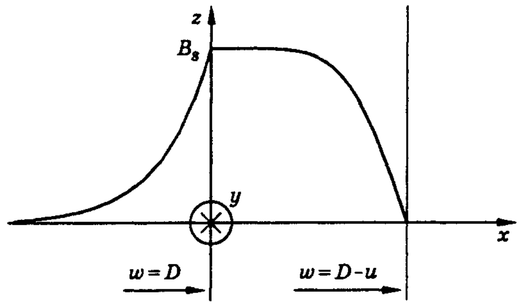

Fig. 1

as metals, the equilibrium between the electron and ionic densities is established practically instantaneously within 10^{-13}–10^{-15} sec and can be ignored.

The second result of the problem is that a negligible current with linear density $\Delta i_w \approx (2/(1+s))(1+s/n^3)(D^2/c^2)i_s$ is trapped in the zone of electron-density oscillations. Although this current is very small, one can show that precisely this current causes the appearance of electric charges on the conductor surfaces parallel to the magnetic field and the appearance of the related electric field $\mathbf{E} = -(1/c)[\mathbf{u} \times \mathbf{B}]$, which compensates for the induction e.m.f. generated under the motion of the conductor with mass velocity $\mathbf{u}$ in the magnetic field.

The current component related to the electron-density oscillations in the shock wave has practically no effect on the magnetic-field distribution. For an ideal shock of zeroth thickness, this implies that the entire current generated under stationary wave propagation is concentrated ahead of the hydrodynamic discontinuity in the diffusion region.

Nonstationary Problem of the Formation of a Shock-Wave Current Zone. Let us consider the problem of the formation of a current zone when a shock wave enters a conducting specimen. We confine ourselves to the simplest plane formulation. We use the reference system related to the shock front which is assumed to be an ideal discontinuity. The material having density ρ_0 and velocity $w = D$ runs against the discontinuity from the left. After the material leaves the discontinuity, the density instantaneously becomes ρ_f and the velocity decreases to $w = (D-u)$. The orientation of the coordinate axes relative to the material flow is shown in Fig. 1. The x axis is directed along the flow velocity $\mathbf{w}$. The magnetic field B is perpendicular to the flow along the z axis, and the electric field is directed along the y axis. The interaction between the magnetic field and the conducting-material flow causes distributions of the electric and magnetic fields and of the current density. Figure 1 shows the distribution of the magnetic field at a certain moment of time.

In a quasi-stationary approximation, the problem reduces to the solution of the magnetic-field diffusion equation [2]

$$\frac{\partial B}{\partial t} + \frac{\partial (wB)}{\partial x} = \frac{c^2}{4\pi}\frac{\partial}{\partial x}\Big(\frac{1}{\sigma}\frac{\partial B}{\partial x}\Big),$$

subject to the boundary conditions

$$B(t,x)_{x\to-\infty} \to B_0, \qquad B(t,x)_{x=(D-u)t} = B_0.$$

In this case, the current density is defined by the Maxwell equation without displacement currents

$$j = -\frac{c}{4\pi}\frac{\partial B}{\partial x},$$

and the electric field is given by Ohm's law for a moving conductor

$$E = \frac{w}{c}B + \frac{1}{\sigma}j.$$

Let us treat the shock wave as an ideal discontinuity of zeroth thickness. In this formulation, the

problem is separated into two problems:
— the problem of field diffusion in the incident flow from the $B_s(t)$ formed when the material is compressed at the shock front to B_0 at infinity from the left;
— the problem of field relaxation behind the shock from $B_s(t)$ to B_0 on the specimen surface moving with a velocity $(D-u)$ from the shock front to the right.

Since an ideal structureless hydrodynamic discontinuity is unable to trap any noticeable current, the magnetic field at the shock itself should be considered continuous.

It is clear from the physical nature of the problem that the magnetic field at the wave front increases under compression. This means that a current of the corresponding direction is generated ahead of the shock. Since, on the specimen surface, the field takes the initial value, the current generated near the surface is equal in magnitude to that formed ahead of the shock but is opposite in direction. Thus, the shock-wave propagation in the conductor is accompanied by the appearance of a system of two currents which are equal in magnitude and opposite in direction and are concentrated in the region ahead of the wave front and near the specimen surface. These currents are called the current and the countercurrent.

Initial Conditions of the Problem. Dividing the problem into the two above-mentioned problems of determination of the upstream and downstream fields, we formulate the initial conditions for these problems at the boundary $x=0$. Apparently one could simply assume that $B_s(0)=B_0$ at the initial moment and then consider the evolution of the field during compression of the material. Such an approach is possible for waves of finite thickness wave. But for an ideal discontinuity, compression occurs instantaneously, and, hence, the field evolution during compression should be treated with caution.

Repeating the well-known calculations for a deformed particle with given mass δm, one can obtain the general result

$$\frac{d}{dt}(\delta\varphi)=\delta m\,\frac{d}{dt}\left(\frac{B}{\rho}\right), \tag{3}$$

where $\delta\varphi$ is the magnetic flux related to this particle. On the other hand, the law of electromagnetic induction for the region occupied by the same particle, combined with Ohm's law, leads to the relation

$$\frac{1}{c}\,\frac{d}{dt}(\delta\varphi)=\frac{j_1}{\sigma_1}-\frac{j_2}{\sigma_2}, \tag{4}$$

in which the densities of the currents and electrical conductivities are determined at the particle boundaries.

At the initial time, there are no currents beyond the region occupied by the wave. Hence, it follows from (4) that for an ideal discontinuity, the condition of flux conservation is satisfied at this moment, and, in accordance with (3), the magnetic field governed by the condition of "freezing" in the material appears at the shock: $B_f=nB_0$ i.e., from the initial moment, it immediately becomes equal to the stationary value for the region behind the shock. Thus, the problem of the distribution of the electromagnetic field and the currents generated when a structureless shock wave enters the conductor with a transverse magnetic field reduces to the solution of two problems of magnetic-field diffusion into a moving conductor with different upstream and downstream velocities and electrical conductivities and the same stationary boundary condition $B(t,0)=nB_0$.

The solution of the problem stated can be visualized as follows. On the conductor surface, a current and a countercurrent are inserted into one another, whose values are determined from (1). The shock wave entering the specimen through this surface separates the current from the countercurrent and entrains the former. In this case, a constant magnetic field $B_f=nB_0$ appears at the wave front, while a diffusion zone with a corresponding current distribution in it is formed ahead of the front because of the field diffusion for a certain time. In the conductor encompassed in wave motion, the field relaxes to the surface to take the initial value B_0 in it, and the countercurrent diffuses slowly from the specimen surface to the material moving behind the wave front.

Below, we give a solution of the problem stated. All formulas are written in dimensionless variables in which the linear scale is referred to the diffusion thickness of the current layer l_0, the time is referred to l_0/D, the magnetic field to B_0, the electric field to $(D/c)B_0$, and the current density to $(c/4\pi)B_0/l_0$.

Fields and Currents Ahead of the Shock. For the region ahead of the jump, we solve the problem by the Laplace transform with respect to time and obtain

$$B(t,x) = 1 + \frac{2(n-1)}{\sqrt{\pi}} \exp(x) \int\limits_{\sqrt{x^2/4t}}^{\infty} \exp\left(-z^2\left(1+\frac{x}{4z^2}\right)^2\right) dz. \tag{5}$$

The current density and the electric field are defined by the relations

$$j = -\frac{\partial B}{\partial x}, \qquad E = B - \frac{\partial B}{\partial x},$$

and, with allowance for (5), we have

$$j(t,x) = \frac{e^x(n-1)}{\sqrt{\pi}} \int\limits_{\sqrt{x^2/4t}}^{\infty} \left(\frac{x}{2z^2}-1\right) \exp\left(-z^2\left(1+\frac{x}{4z^2}\right)^2\right) dz - \frac{e^{(x/2-t/4)}}{\sqrt{\pi t}} e^{-x^2/4t}; \tag{6}$$

$$E(t,x) = 1 + \frac{e^x(n-1)}{\sqrt{\pi}} \int\limits_{\sqrt{x^2/4t}}^{\infty} \left(1+\frac{x}{4z^2}\right) \exp\left(-z^2\left(1+\frac{x}{4z^2}\right)^2\right) dz - \frac{e^{(x/2-t/4)}}{\sqrt{\pi t}} e^{-x^2/4t}. \tag{7}$$

We emphasize that all quantities ahead of the shock depend only on the compressibility n.

The asymptotic distributions of fields and currents (5)–(7) for $t \ll 1$ and $|x| < t$ are of the form

$$\begin{gathered} B(t,x) \simeq n + (n-1)\frac{x}{2} + (n-1)\frac{x}{\sqrt{\pi t}} + \frac{3}{4}(n-1)x\sqrt{t/\pi}, \\ j(t,x) \simeq -\frac{n-1}{2} - \frac{n-1}{\sqrt{\pi t}} - \frac{3(n-1)}{4}\sqrt{t/\pi}, \\ E(t,x) \simeq \frac{n+1}{2} + (n-1)\frac{x}{2} + (n-1)\frac{x-1}{\sqrt{\pi t}} + \frac{3}{4}(n-1)(x-1)\sqrt{t/\pi}. \end{gathered} \tag{8}$$

For large times and, as previously, for $|x| < t$, they are simplified to

$$B(t,x) \simeq 1 + (n-1)e^x, \qquad j(t,x) \simeq -(n-1)e^x, \qquad E(t,x) \simeq 1 \tag{9}$$

and become stationary. We note that in these formulas, $x < 0$.

Fields and Currents Behind the Shock. A solution of this part of the problem can be obtained by solving the problem of the diffusion of a given current from the specimen surface to the conducting region formed by a shock wave that transforms the compressed nonconducting material to a conducting state [3]. Upon substitution of variables and necessary recalculation for the new frame of reference, for the field behind the shock $[0 < x \leqslant (t/n)]$, we obtain

$$B(t,x) = 1 + (n-1)\left(1 - \frac{2}{\sqrt{\pi}} \exp\left(-\frac{s(x-t/n)^2}{4t}\right) \int\limits_0^{\infty} \exp(-z^2) \frac{\sinh(zx\sqrt{s}/\sqrt{t})}{\sinh(z\sqrt{st/n})} dz\right). \tag{10}$$

The current density and the electric field behind the shock are defined by the relations

$$j = -\frac{\partial B}{\partial x}, \qquad E = \frac{B}{n} - \frac{1}{s}\frac{\partial B}{\partial x},$$

and are dependent on the compression of the material n and on the jump of electrical conductivity s:

$$j(t,x) = \frac{2(n-1)}{\sqrt{\pi}} \exp\left(-\frac{s(x-t/n)^2}{4t}\right)$$

$$\times\left(\sqrt{s/t}\int\limits_0^{\infty} z\exp(-z^2)\frac{\cosh(zx\sqrt{s/t})}{\sinh(z\sqrt{st/n})} dz - \frac{s}{2}\left(\frac{x}{t}-\frac{1}{n}\right)\int\limits_0^{\infty} \exp(-z^2)\frac{\sinh(zx\sqrt{s/t})}{\sinh(z\sqrt{st/n})} dz\right); \tag{11}$$

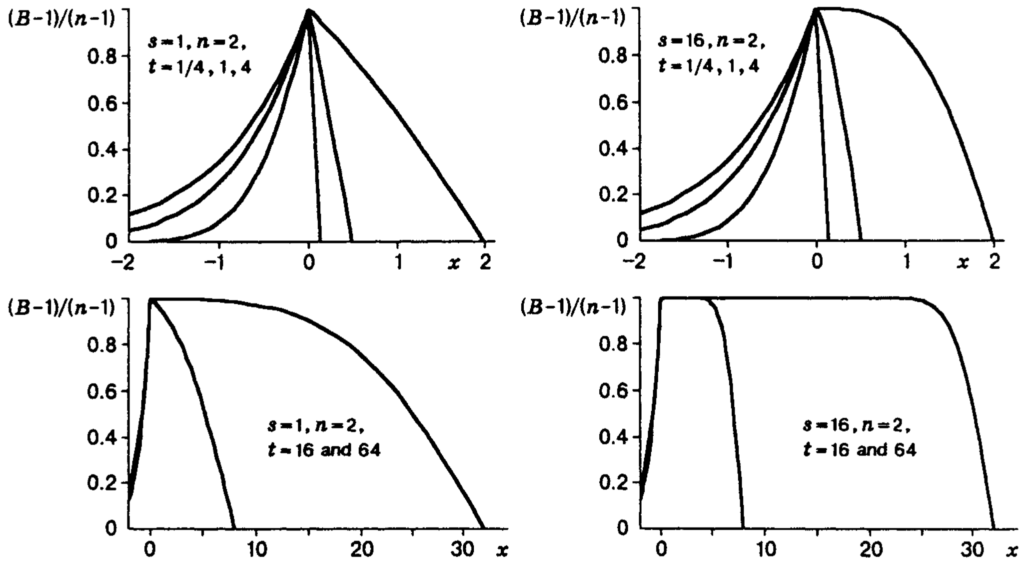

Fig. 2

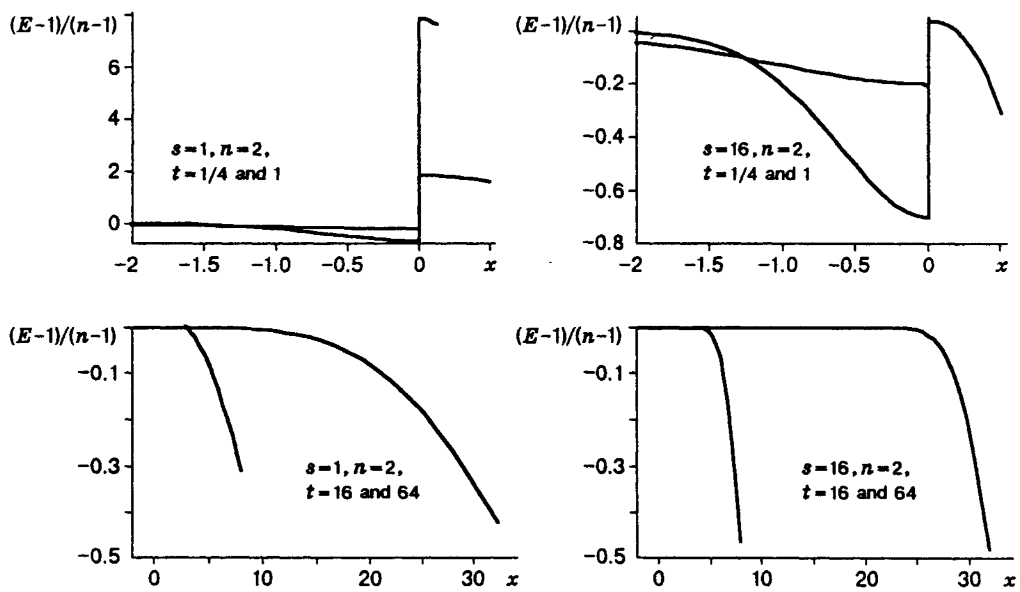

Fig. 3

$$E(t,x) = 1 + \frac{2(n-1)}{\sqrt{\pi}} \exp\Big(-\frac{s(x-t/n)^2}{4t}\Big)$$

$$\times\Big(\frac{1}{\sqrt{st}}\int_0^\infty z\exp(-z^2)\frac{\cosh(zx\sqrt{s/t})}{\sinh(z\sqrt{st}/n)}\,dz - \frac{1}{2}\Big(\frac{x}{t}+\frac{1}{n}\Big)\int_0^\infty \exp(-z^2)\frac{\sinh(zx\sqrt{s/t})}{\sinh(z\sqrt{st}/n)}\,dz\Big). \qquad (12)$$

The asymptotic distributions (10)–(12) for $t \ll 1$ near the specimen surface ($x < t/n$ and $x \simeq t/n$), where the current density and the electric field are maximal, are of the form

$$B(t,x) \simeq n - (n-1)\frac{x}{t/n}, \quad j(t,x) \simeq \frac{n-1}{t/n}, \quad E(t,x) \simeq 1 - (n-1)\frac{x}{t/n} + (n-1)\frac{1}{st/n}. \qquad (13)$$

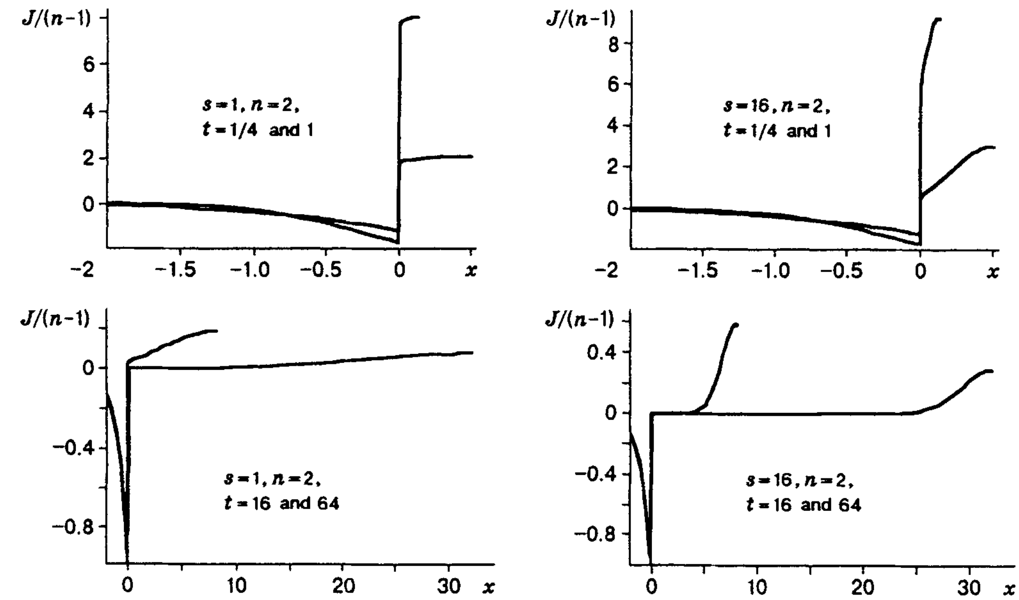

Fig. 4

For large times, they become

$$B(t,x) \simeq 1 - (n-1)\Big(x - \frac{t}{n}\Big)\sqrt{\frac{s}{\pi t}}, \qquad j(t,x) \simeq (n-1)\sqrt{\frac{s}{\pi t}},$$
$$E(t,x) \simeq \frac{1}{n} + (n-1)\sqrt{\frac{s}{\pi t}} - \frac{n-1}{n}\Big(x - \frac{t}{n}\Big)\sqrt{\frac{s}{\pi t}}. \tag{14}$$

Analysis and Discussion of the Results. First, we note that the distributions of fields and currents ahead of the shock-wave front is absolutely independent of the conductivity of the material behind the jump. This is clear: the conducting material shields everything that appears behind the wave front. Only the current magnitude is determined by the compression of the magnetic field and of the material. The current diffusion in the incident flow first proceeds fairly rapidly: $\propto 1/t$. The asymptotic relations (8) and (9) show that the distributions of fields and currents ahead of the wave monotonically become stationary with time. It is noteworthy that solution (5) reaches the stationary distribution of a magnetic field presented in [1] and, as can be shown, this occurs fairly rapidly, over the period of time during which the wave travels only several thicknesses of the diffusion current layer.

From the asymptotic relations (13) and (14) we can see that the distributions of fields and currents behind the wave changes radically with time. The current is initially almost uniform and simply fills the entire conducting material behind the wave. In this case, the magnetic field decreases almost linearly from its "frozen" value n to 1 on the specimen surface. The electric field changes most rapidly: its value $\propto 1/t$. The change in electrical conductivity has practically no effect on the current and magnetic-field distributions at the initial time. But for the electric field, the electrical-conductivity value is governing ($E \propto 1/s$), and the electric field behind the wave decreases sharply with an increase in conductivity. With time, the distributions of fields and currents behind the wave is transformed to the classical distribution of these quantities in an ordinary skin-layer: all quantities appear to be $\propto \sqrt{s/t}$.

The graphs in Figs. 2-4, which give the calculation results obtained in this work for the electromagnetic-field and current-density distributions, give detailed dynamics of fields and currents under shock-wave propagation in a conducting specimen. The calculations were carried out for a material with compressibility $n = 2$ for times $t = 1/4$, 1, 16, and 16. To illustrate the dependence on the change in electrical conductivity in the shock wave, we considered two cases: the electrical conductivity is constant ($s = 1$) and increases by a factor of 16 behind the shock ($s = 16$). The vertical scale for the electric field and the current density is different in different graphs in Figs. 3 and 4. To compare the results with different electrical conductivities, one should take into account that for the same times, the field–current distributions ahead of the shock wave are the same and do not depend on the electrical-conductivity jump in the wave.

It is evident from the graphs that the numerical-calculation results are in agreement with the qualitative conclusions derived from the analysis of the asymptotics. The stationary electromagnetic pattern ahead of the ideal hydrodynamic shock is formed only for four diffusion sizes.

This work was supported by the International Science Foundation and Russian Foundation for Fundamental Research (Joint grant No. RBO 300).

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